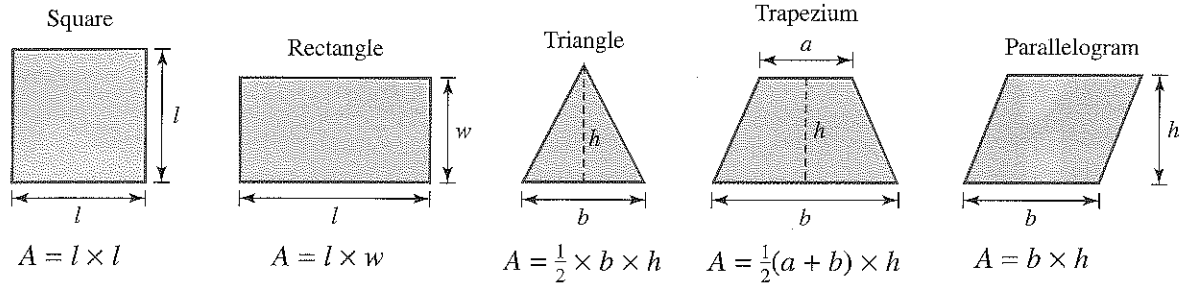# Area

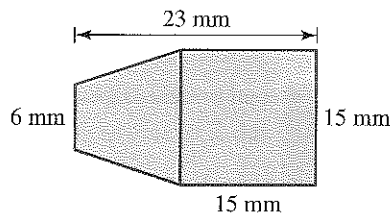## KEY CONCEPTS

➤ The area of a two-dimensional (2-D) shape is the amount of space enclosed by that shape.

➤ Area is measured using square units such as mm^2, cm^2, m^2 and km^2.

➤ The areas of 2-D shapes can be calculated using formulae.

Square	Rectangle	Triangle	Trapezium	Parallelogram
$A = l \times l$	$A = l \times w$	$A = \frac{1}{2} \times b \times h$	$A = \frac{1}{2}(a + b) \times h$	$A = b \times h$

EXAMPLE 1

Calculate the area of the following shape.

23 mm

6 mm   15 mm

15 mm

THINK

① WRITE

① This composite shape is made up of a trapezium and a square. Therefore, the total area is equal to the area of the trapezium plus the area of the square.

$A_{total} = A_{trapezium} + A_{square}$

② Write out the formula for calculating the area of a trapezium.

$A_{trapezium} = \frac{1}{2}(a + b) \times h$

③ Identify the values of a, b and h. Substitute them into the formula. Calculate your answer.

$a = 6$
$b = 15$
$h = 23 - 15 = 8$
$A_{trapezium} = \frac{1}{2}(6 + 15) \times 8$
$A_{trapezium} = 84\ mm^2$

④ Write out the formula for calculating the area of a square.

$A_{square} = l \times l$

⑤ Identify the value of l and substitute it into the formula. Calculate your answer.

$A_{square} = l \times l$
$= 15 \times 15$
$= 225\ mm^2$

⑥ Add the area of the trapezium and the area of the square to find the total area. Include appropriate units in your answer.
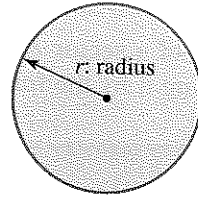
$A_{total} = A_{trapezium} + A_{square}$
$= 84 + 225$
$= 309\ mm^2$

Area of a circle

The area of a circle can be calculated using the formula:
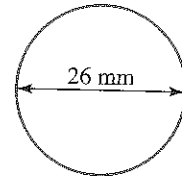
$A_{circle} = \pi \times r^2$, where
r = radius of the circle
π (pi) = the ratio of the circumference to the diameter; it is approximately equal to 3.141 59 or $\frac{22}{7}$.

r: radius

EXAMPLE 2

Calculate the area of the following circle. Give your answer correct to 2 decimal places.

26 mm

THINK

❶ The area of the circle is the space enclosed within the shape. Write the formula for calculating the area of a circle.

❷ Substitute the value of the radius into the formula and calculate the area.

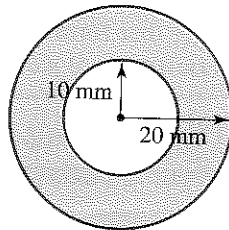❸ Give your answer correct to 2 decimal places and include the appropriate units.

WRITE

$A = \pi \times r^2$

$A = \pi \times r^2$
$= \pi \times (26/2)^2$
$= 169\,\pi$

$A = 531.14\,mm^2$

EXAMPLE 3

Calculate the area of the shaded region.

10 mm

20 mm

THINK

❶ The shape is made up of 2 circles. The shaded area can be found by finding the area of the larger circle and subtracting the area of the smaller circle.

❷ Calculate the area of the larger circle with a radius of 20 mm.

❸ Calculate the area of the smaller circle with a radius of 10 mm.

❹ Calculate the area of the shaded region.

WRITE

$A_{shaded\ region} = A_{large\ circle} - A_{small\ circle}$

$A_{large\ circle} = \pi \times 20^2$
$= 400\,\pi$

$A_{small\ circle} = \pi \times 10^2$
$= 100\pi$

$A_{shaded\ region} = 400\pi - 100\pi$
$= 300\pi\ (942.48)\,mm^2$

Area of a sector

➤ A sector of a circle is a part of a circle shaped like a slice of pie. It is bounded by two radii and an arc.

➤ The area of a sector can be calculated by finding the area of the circle and multiplying it by the fraction of the angle that the sector forms at the centre of the circle.
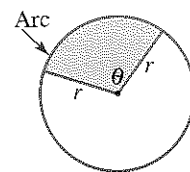
> Area (sector) $= \pi \times r^2 \times \dfrac{\theta}{360}$, where
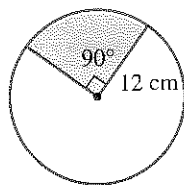> r = radius of the circle
> θ = angle that the sector makes with the centre of the circle in degrees.

$$A = \pi r^2 \times \frac{\theta}{360}$$

EXAMPLE 4

Calculate the area of the shaded region.

THINK	WRITE
① Write out the formula for calculating the area of a sector.	$A_{sector} = \pi \times r^2 \times \dfrac{\theta}{360}$
② Substitute the values into the formula. Calculate the area of the shaded region by multiplying the area of the complete circle by the fraction of the angle that forms the sector.	$A_{sector} = \pi \times 12^2 \times \dfrac{\theta}{360}$ $= 144\pi \times \frac{90}{360}$ $= 133.10$
③ Include the appropriate units in your final answer.	$A_{sector} = 113.10 \text{ cm}^2$

LEARNING EXPERIENCE

Counting squares

Equipment: grid paper, ruler, circle template

1 On the grid paper, draw an example of each of the following shapes: square; rectangle; triangle; trapezium; circle.

2 Estimate the area of each of your shapes (in cm²) by counting the squares.

3 Share your drawings with a partner and ask them to calculate the area of each shape by using the appropriate formula.

4 Compare your results.

EXERCISE 12B

Now try these

1 Write the definition of *area* in your own words.

2 **MULTIPLE CHOICE** The area of a rectangle of size $12\,cm \times 14\,cm$ would be equal to:
A $168\,cm^2$
B $52\,cm^2$
C $186\,cm^2$
D $26\,cm^2$
E $52\,mm^2$

3 **EXAMPLE 1** Calculate the areas of the following shapes.

a

9 m
7 m
6 m

b

12 mm
11 mm
16 mm

c

14 cm
18 cm
17 cm

d

22 mm
12 mm
22 mm

e

45 cm
7 cm
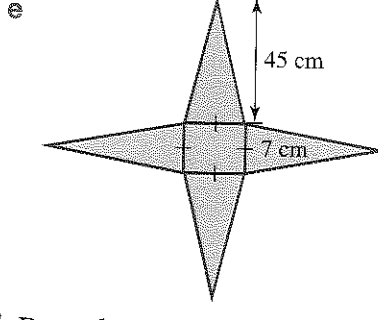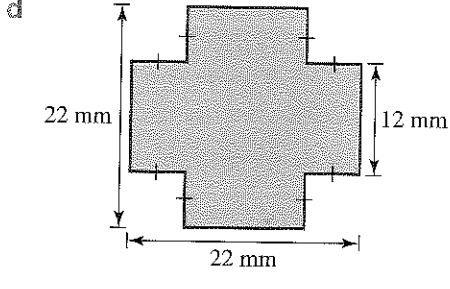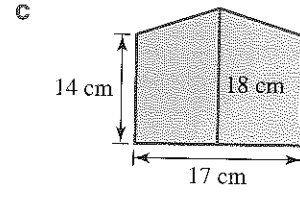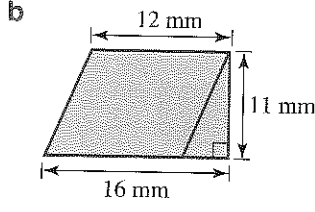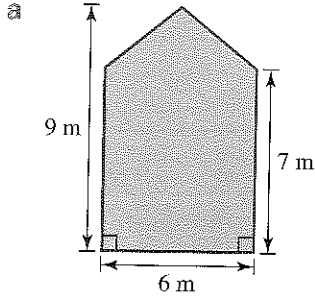
4 The area of an unknown shape is $64\,cm^2$. Draw three different labelled shapes that have an area of $64\,cm^2$.

5 **EXAMPLE 2** Calculate the shaded area in each of the following shapes. Give your answers correct to 2 decimal places.

a

12 mm

b

34 cm

c

12 mm
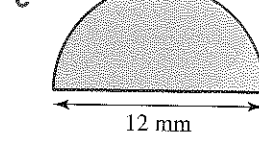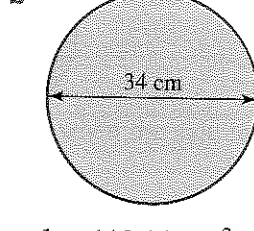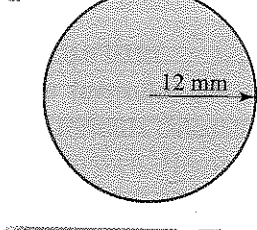
6 **MULTIPLE CHOICE** The area of a circle is equal to $113.14\,mm^2$. The radius of the circle is closest to:
A 36 mm
B 6 mm
C 2133 mm
D 7 mm
E 60 mm

12-C
Total surface area

KEY CONCEPTS

- The total surface area (TSA) of a 3-D object is the total area of each face of that object.
- The total surface area of a 3-D object can be calculated by finding the area of each individual face and adding the areas together.
- Drawing the net of the solid can help you to determine the shapes of the individual faces that make up the solid.
- Total surface area is measured using square units appropriate to the task (mm^2, cm^2, m^2 or km^2).
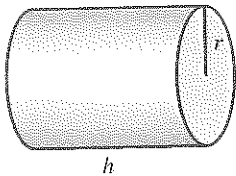- For some 3-D objects the total surface area can be calculated using an appropriate formula.
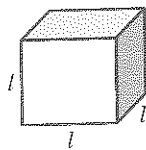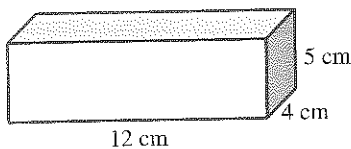
$$TSA_{cube} = 6 \times l^2$$
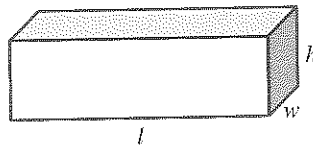
$$TSA_{rectangular\ prism} = 2(lw + wh + lh)$$

$$TSA_{cylinder} = 2\pi rh + 2\pi r^2$$

EXAMPLE 1

Calculate the total surface area of the following rectangular prism.

5 cm
4 cm
12 cm

THINK	WRITE
1 The rectangular prism is made up of 6 rectangular faces. The area of each of these faces can be calculated by multiplying the length by the width.	$TSA_{rectangular\ prism} = 2(lw + wh + lh)$
2 Identify the pronumerals and substitute them into the formula to calculate the TSA.	$TSA_{rectangular\ prism} = 2(5 \times 4 + 4 \times 12 + 5 \times 12)$ $= 2(20 + 48 + 60)$ $= 256$
3 Include the appropriate units in your final answer.	$TSA = 256\,cm^2$

3 EXAMPLE 1 Calculate the total surface area of each of the following objects.

a

12 mm

3 mm
4 mm

b

10 cm

22 cm

c

3.5 cm

3.5 cm

3.5 cm

d

1.1 m

0.9 m
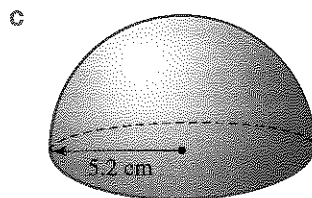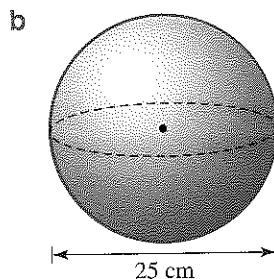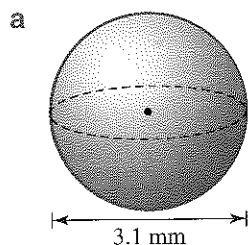
3.2 m

e

50 cm

65 cm
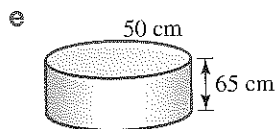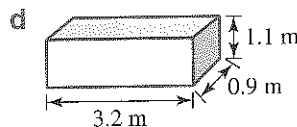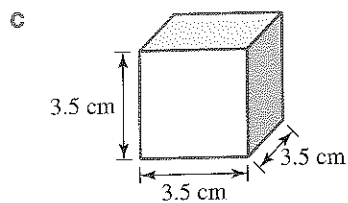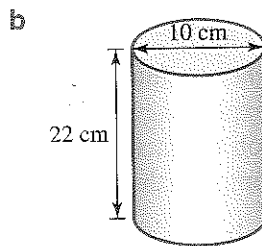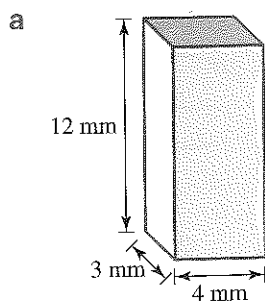
4 Design, draw and label a cube that has a total surface area of $150\,\text{cm}^2$.

5 a Design, draw and label a 3-D object that has a total surface area of $220\,\text{m}^2$.

b Draw and label a different 3-D object that also has a total surface area of $220\,\text{m}^2$.

6 EXAMPLE 2 Calculate the total surface area of the following objects.

a

3.1 mm

b

25 cm

c

5.2 cm

7 MULTIPLE CHOICE A sphere has a total surface area of $125\,\text{mm}^2$. The radius of the sphere correct to 2 decimal places is:

A 3.15 mm

B 9.94 mm

C 31.5 mm

D 31.25 mm

E 15.71 mm